Calculus Warm Up #5-5

Use the first and/or second derivative to find and confirm any extrema for the graph of

$$f(x) = x^{2/3} - 3$$

Classwork Table for investigating the function:

Places of interest include x-values where the first or second derivative = 0 or is undefined.

$x = 0$
$x = 2$
$x = 3$

Conclusion column is for extrema, points of inflection, function increasing or decreasing, and concavity

3

$f(x) = x^4 - 4x^3$ $f'(x) = 4x^2(x-3)$ $f''(x) = 12x(x-2)$

| Places of interest and intervals between | test #'s on the interval | f′(x) +, - or 0 | f″(x) +, - or 0 | Conclusion |
|---|---|---|---|---|
| $(-\infty, 0)$ | -1 | − | + | f decr. concave up |
| $x = 0$ | — | 0 | 0 | PI (0, 0) |
| $(0, 2)$ | 1 | − | − | f decr. conc down |
| $x = 2$ | — | | 0 | PI (2, -16) |
| $(2, 3)$ | 2.5 | − | + | f decr. conc up |
| $x = 3$ | — | 0 | + | Min (3, -27) |
| $(3, \infty)$ | 4 | + | + | f incr; conc up |

$f(x) = x^4 - 4x^3$

Intercepts: $(0,0)$ $(4,0)$

Asymptotes: $\emptyset$

End behavior: both ends up.

$f'(x)$: $= 4x^3 - 12x^2$

Critical #'s: $x = 0, 3$

$f''(x)$: $= 12x^2 - 24x$

$f''(x) = 0$

$x = 0, 2$

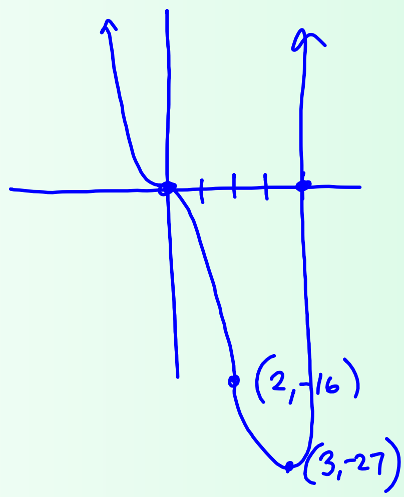
Make a table.
use $x = 0, 2, 3$ and
the intervals between

Graph sketching:

1. Find the intercepts, asymptotes and check end behavior for what to expect. (Use your Precalc tools!)

2. Use the first derivative and critical numbers to find where the graph is increasing, decreasing or has possible extrema.

3. Use the second derivative to determine shape (concavity).




# Groups:

Go over #23 & 25.

1) Compare first and second derivatives. Fix any disagreements.

2) Compare critical #'s and intervals in your table. Fix any disagreements.

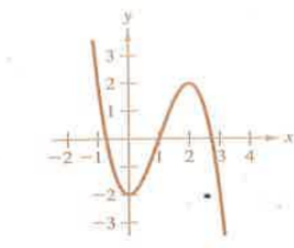
3) Compare the conclusions and fix any disagreements.

4) Compare graphs for details.

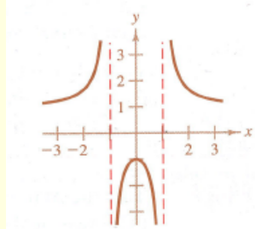
HW Questions: p. 180

In Exercises 1–6, find the open intervals on which the graph of the given function is concave upward and those on which it is concave downward.

**3.** $y = -x^3 + 3x^2 - 2$



**5.** $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$



In Exercises 7–18, identify all relative extrema. Use the Second Derivative Test where applicable.

7. $f(x) = 6x - x^2$

**9.** $f(x) = (x - 5)^2$

11. $f(x) = x^3 - 3x^2 + 3$

15. $f(x) = x^{2/3} - 3$

In Exercises 19–34, sketch the graph of the given function and identify all relative extrema and points of inflection.

23. $f(x) = \frac{1}{4}x^4 - 2x^2$

| Places of interest and intervals between | test #'s on the interval | f´(x) +, - or 0 | f˝(x) +, - or 0 | Conclusion |
|---|---|---|---|---|
| | | | | |

In Exercises 19–34, sketch the graph of the given function and identify all relative extrema and points of inflection.

Intercepts: (0,0) & (4,0)
End beh. both sides rising.

25. $f(x) = x(x-4)^3$

$f'(x) = 4(x-4)^2(x-1) \rightarrow f'(x)=0$ @ $x=4,1$

$f''(x) = 12(x-4)(x-2) \rightarrow f''(x)=0$ @ $x=4,2$

| Places of interest and intervals between | test #'s on the interval | f´(x) +, - or 0 | f˝(x) +, - or 0 | Conclusion |
|---|---|---|---|---|
| $(-\infty, 1)$ | 0 | − | + | f decr., Concave Up |
| $x=1$ | | 0 | + | Min @ (1,-27) |
| $(1,2)$ | 1.5 | + | + | f incr., Concave Up |
| $x=2$ | | | 0 | PI @ (2,-16) |
| $(2,4)$ | 3 | + | − | f incr., Concave Dwn |
| $x=4$ | | 0 | 0 | PI @ (4,0) |
| $(4,\infty)$ | 5 | + | + | f incr., Concave Up |

29. $f(x) = x\sqrt{x+3}$

Domain: $x \geq -3$

Intercepts: $(-3,0)$ & $(0,0)$

$$f'(x) = \frac{3x+6}{2\sqrt{x+3}}$$

$$f''(x) = \frac{3x+12}{4(x+3)\sqrt{x+3}}$$

| Places of interest and intervals between | test #'s on the interval | f´(x) +, - or 0 | f˝(x) +, - or 0 | Conclusion |
|---|---|---|---|---|
| | | | | |

29. $f(x) = x\sqrt{x+3}$

Domain: $x \geq -3$

Intercepts: $(-3,0)$ & $(0,0)$

$$f'(x) = \frac{3x+6}{2\sqrt{x+3}} \rightarrow f'(x) = 0 \text{ @ } x=-2\text{, undef @ } x=-3$$

$$f''(x) = \frac{3x+12}{4(x+3)\sqrt{x+3}} \rightarrow f''(x) = 0 \text{ @ } x=-4\text{, undef. @ } x=-3$$

(but $f(-4)$ is undef.)

| Places of interest and intervals between | test #'s on the interval | f´(x) +, - or 0 | f˝(x) +, - or 0 | Conclusion |
|---|---|---|---|---|
| $x=-3$ | | undef | undef | Endpt @ $(-3,0)$ |
| $(-3,-2)$ | $-2.5$ | $-$ | $+$ | f decr., Concave Up |
| $x=-2$ | | 0 | $+$ | Min @ $(-2,-2)$ |
| $(-2,\infty)$ | 0 | $+$ | $+$ | f incr., Concave Up |

# 4.5

-Limits at infinity (as $x \to \infty$)

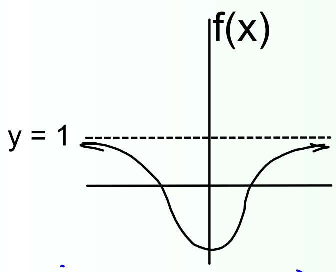
-Horizontal asymptotes

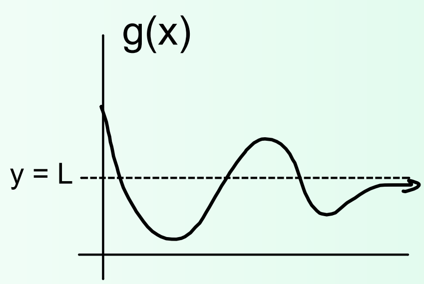
Remember Limit Properties:

$$\lim_{x\to\infty} [f(x) + g(x)] = \lim_{x\to\infty} f(x) + \lim_{x\to\infty} g(x)$$

$$\lim_{x\to\infty} [f(x)\, g(x)] = \left(\lim_{x\to\infty} f(x)\right)\left(\lim_{x\to\infty} g(x)\right)$$

## Horizontal Asymptotes:




$\lim_{x\to -\infty} f(x) = 1$ $\lim_{x\to\infty} f(x) = 1$ $\lim_{x\to\infty} f(x) = L$

Both fit the Definition of a Horizontal Asymptote (p. 183)

Characteristics of Horizontal Asymptotes to consider and compare to vertical asymptotes:

1) Limit exists, it = L, a real number
2) Function can be defined on y = L.
3) There can be at most 2 horizontal asymptotes.

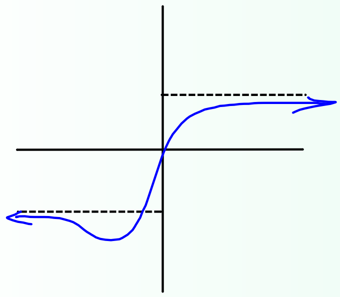
**DEFINITION OF HORIZONTAL ASYMPTOTE**

If

$$\lim_{x\to-\infty} f(x) = L \quad \text{or} \quad \lim_{x\to\infty} f(x) = L$$

then the line $y = L$ is called a **horizontal asymptote** of the graph of $f$.

Example of 2 Horizontal Asymptotes:



## Limits at Infinity:

$$\lim_{x\to\infty} \frac{c}{x^r} = 0$$

c = real # constant
r = positive, rational #

Examples:

1. $\lim_{x\to\infty} \frac{3}{x^2}$

$= \frac{3}{\infty}$

$= 0$

2. $\lim_{x\to\infty} \frac{-5}{\sqrt{x}}$

$-\frac{5}{\infty}$

$0$

Evaluate:

$$\lim_{x\to\infty} \frac{2x - 1}{x + 1} \longrightarrow$$ Needs fussing. $\cdot \dfrac{\frac{1}{x}}{\frac{1}{x}}$

$$= \frac{2(\infty) - 1}{\infty + 1}$$

$$= \frac{\infty}{\infty}$$

limit can not be determined.

$$\lim_{x\to\infty} \frac{\frac{2x}{x} - \frac{1}{x}}{\frac{x}{x} + \frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{2 - \frac{1}{x}}{1 + \frac{1}{x}}$$

$$\frac{2 - 0}{1 + 0}$$

$$\boxed{2}$$

Find the following limits.

(a) $\lim\limits_{x\to\infty} \dfrac{2x + 5}{3x^2 + 1} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}$ (b) $\lim\limits_{x\to\infty} \dfrac{2x^2 + 5}{3x^2 + 1}$ (c) $\lim\limits_{x\to\infty} \dfrac{2x^3 + 5}{3x^2 + 1}$

$$= \lim_{x\to\infty} \frac{\frac{2\cancel{x}}{x^{\cancel{2}}} + \frac{5}{x^2}}{\frac{3\cancel{x^2}}{\cancel{x^2}} + \frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x} + \frac{5}{x^2}}{3 + \frac{1}{x^2}}$$

$$= \frac{0 + 0}{3 + 0}$$

$$= \boxed{0}$$

Find the following limits.

(a) $\lim_{x\to\infty} \dfrac{2x+5}{3x^2+1}$ (b) $\lim_{x\to\infty} \dfrac{2x^2+5}{3x^2+1}$ (c) $\lim_{x\to\infty} \dfrac{2x^3+5}{3x^2+1}$

$$= \lim_{x\to\infty} \frac{\frac{2x}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{5}{x^2}}{3+\frac{1}{x^2}}$$

$$= \frac{0+0}{3+0}$$

$$= \boxed{0}$$

$$\lim_{x\to\infty} \frac{\frac{2\cancel{x^2}}{\cancel{x^2}}+\frac{5}{x^2}}{\frac{3\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}}$$

$$\lim_{x\to\infty} \frac{2+\frac{5}{x^2}}{3+\frac{1}{x^2}}$$

$$\frac{2+0}{3+0}$$

$$\boxed{\frac{2}{3}}$$

Find the following limits.

(a) $\lim_{x\to\infty} \dfrac{2x+5}{3x^2+1}$ (b) $\lim_{x\to\infty} \dfrac{2x^2+5}{3x^2+1}$ (c) $\lim_{x\to\infty} \dfrac{2x^3+5}{3x^2+1}$

$$= \lim_{x\to\infty} \frac{\frac{2x}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{5}{x^2}}{3+\frac{1}{x^2}}$$

$$= \frac{0+0}{3+0}$$

$$= \boxed{0}$$

$$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{1}{x^2}}$$

$$\lim_{x\to\infty} \frac{2+\frac{5}{x^2}}{3+\frac{1}{x^2}}$$

$$\frac{2+0}{3+0}$$

$$\boxed{\frac{2}{3}}$$

$$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3}+\frac{5}{x^3}}{\frac{3x^2}{x^3}+\frac{1}{x^3}}$$

$$\lim_{x\to\infty} \frac{2+\frac{5}{x^3}}{\frac{3}{x}+\frac{1}{x^3}}$$

$$\frac{2+0}{0+0}$$

$$\frac{2}{0}$$

DNE

Determine the following limits.

(a) $\lim\limits_{x \to \infty} \dfrac{3x - 2}{\sqrt{2x^2 + 1}}$ (b) $\lim\limits_{x \to -\infty} \dfrac{3x - 2}{\sqrt{2x^2 + 1}} \cdot \dfrac{\frac{1}{-x}}{\frac{1}{\sqrt{x^2}}}$

Since x is a negative number as we approach -∞, we need the opposite of x:

let $\sqrt{x^2} = -x$

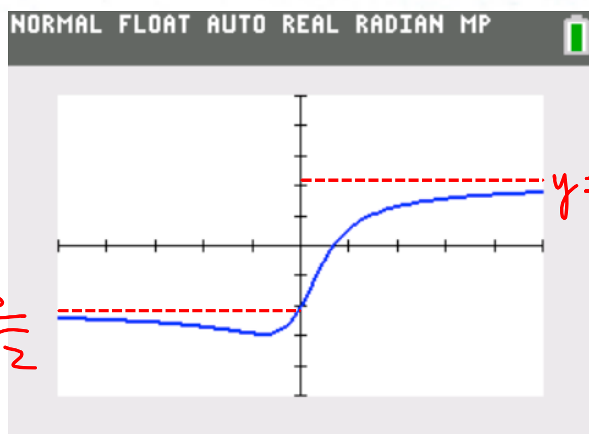
$$\lim_{x \to -\infty} \frac{-\frac{3\cancel{x}}{\cancel{x}} + \frac{2}{x}}{\sqrt{\frac{2\cancel{x^2}}{\cancel{x^2}} + \frac{1}{x^2}}}$$

$$\frac{-3 + 0}{\sqrt{2 + 0}}$$

$$-\frac{3}{\sqrt{2}}$$

Determine the following limits.

(a) $\lim\limits_{x \to \infty} \dfrac{3x - 2}{\sqrt{2x^2 + 1}}$ (b) $\lim\limits_{x \to -\infty} \dfrac{3x - 2}{\sqrt{2x^2 + 1}}$




$y = \dfrac{3}{\sqrt{2}}$

$y = -\dfrac{3}{\sqrt{2}}$

# HW: p. 188 # 9 - 29 odd

(Tuesday HW Quiz pgs. 166, 173, 180, 188)

Quiz: 4.1 - 4.4, Tuesday
Extrema on open and closed intervals
Mean Value Theorem
Increasing/decreasing intervals (1st der. test)
Concavity & 2nd der. test